Lecture 28

Activity-Selection Problem (contd.)

Activity-Selection

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Output: Find a largest-size subset of mutually compatible activities.

Two activities a_i and a_j are mutually compatible if either $s_i \geq f_j$ or $s_j \geq f_i$.

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$S_{i,j}$ = Set of activities that start after activity a_i finishes and that finish before activity a_j starts.

Observation: $S_{i,j}$ can only contains activities from $\{a_{i+1}, a_{i+2}, \ldots, a_{j-1}\}$.

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$$S_{3,9} = \{a_4, a_5, a_7\}$$

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$$S_{5,9} = \varnothing$$

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For what i, j, $A_{i,j}$ will give the desired answer?

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Note: We add two dummy activities a_0 and a_{n+1} at the ends so that $A_{0,n+1}$ is the desired answer.

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$c_{i,j} = |A_{i,j}|$

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We will develop recurrence for $c_{i,j}$ after establishing optimal substructure

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Optimal Substructure in Activity-Selection

Let's try to find $A_{0,12}$ and $c_{0,12}$.

i	0	1	2	3	4	5	6	7	8	9	10	11	12
s_i	0	1	3	0	6	7	5	8	7	10	2	12	16
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Let's try to find $A_{0,12}$ and $c_{0,12}$.

i	0	1	2	3	4	5	6	7	8	9	10	11	12
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If a_6 is part for some $A_{0,12}$, then:

Optimal Substructure in Activity-Selection

Let's try to find $A_{0,12}$ and $c_{0,12}$.

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If a_6 is part for some $A_{0,12}$, then:

- $A_{0,12} =$

Optimal Substructure in Activity-Selection

Let's try to find $A_{0,12}$ and $c_{0,12}$.

i	0	1	2	3	4	5	6	7	8	9	10	11	12
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If a_6 is part for some $A_{0,12}$, then:

- $A_{0,12} =$
- $c_{0,12} =$

Optimal Substructure in Activity-Selection

Let's try to find $A_{0,12}$ and $c_{0,12}$.

i	0	1	2	3	4	5	6	7	8	9	10	11	12
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f_i	0	4	5	6	7	9	10	10	11	12	14	16	16

If a_6 is part for some $A_{0,12}$, then:

- $A_{0,12} = \quad \{a_6\}$
- $c_{0,12} =$

Optimal Substructure in Activity-Selection

Let's try to find $A_{0,12}$ and $c_{0,12}$.

i	0	1	2	3	4	5	6	7	8	9	10	11	12
s_i	0	1	3	0	6	7	5	8	7	10	2	12	16
f_i	0	4	5	6	7	9	10	10	11	12	14	16	16

If a_6 is part for some $A_{0,12}$, then:

- $A_{0,12} = A_{0,6} \cup \{a_6\}$
- $c_{0,12} =$

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Let's try to find $A_{0,12}$ and $c_{0,12}$.

i	0	1	2	3	4	5	6	7	8	9	10	11	12
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If a_6 is part for some $A_{0,12}$, then:

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- $c_{0,12} =$

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Let's try to find $A_{0,12}$ and $c_{0,12}$.

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If a_6 is part for some $A_{0,12}$, then:

- $A_{0,12} = A_{0,6} \cup \{a_6\} \cup A_{6,12}$
- $c_{0,12} =$

Prove by contradiction if $A_{0,12}$ that contains a_6 doesn't contain $A_{0,6}$ or $A_{6,12}$, it is not a maximum set …

Optimal Substructure in Activity-Selection

Let's try to find $A_{0,12}$ and $c_{0,12}$.

i	0	1	2	3	4	5	6	7	8	9	10	11	12
s_i	0	1	3	0	6	7	5	8	7	10	2	12	16
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If a_6 is part for some $A_{0,12}$, then:

- $A_{0,12} = A_{0,6} \cup \{a_6\} \cup A_{6,12}$
- $c_{0,12} = c_{0,6} + c_{6,12} + 1$

Prove by contradiction if $A_{0,12}$ that contains a_6 doesn't contain $A_{0,6}$ or $A_{6,12}$, it is not a maximum set …

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$A_{i,j}$ = A maximum set of mutually compatible activities in $S_{i,j}$ and $c_{i,j} = |A_{i,j}|$

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$A_{i,j}$ = A maximum set of mutually compatible activities in $S_{i,j}$ and $c_{i,j} = |A_{i,j}|$

If $a_{i+1} \in A_{i,j}$, then $c_{i,j} = c_{i,i+1} + c_{i+1,j} + 1$

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If $a_{i+1} \in A_{i,j}$, then $c_{i,j} = c_{i,i+1} + c_{i+1,j} + 1$

If $a_{i+2} \in A_{i,j}$, then $c_{i,j} = c_{i,i+2} + c_{i+2,j} + 1$

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$A_{i,j}$ = A maximum set of mutually compatible activities in $S_{i,j}$ and $c_{i,j} = |A_{i,j}|$

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If $a_{i+2} \in A_{i,j}$, then $c_{i,j} = c_{i,i+2} + c_{i+2,j} + 1$

If $a_{i+3} \in A_{i,j}$, then $c_{i,j} = c_{i,i+3} + c_{i+3,j} + 1$

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If $a_{i+3} \in A_{i,j}$, then $c_{i,j} = c_{i,i+3} + c_{i+3,j} + 1$

$\vdots$

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If $a_{i+3} \in A_{i,j}$, then $c_{i,j} = c_{i,i+3} + c_{i+3,j} + 1$

$\vdots$

If $a_{j-1} \in A_{i,j}$, then $c_{i,j} = c_{i,j-1} + c_{j-1,j} + 1$

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If $a_{i+3} \in A_{i,j}$, then $c_{i,j} = c_{i,i+3} + c_{i+3,j} + 1$

$\vdots$

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Optimal Substructure in Activity-Selection

$A_{i,j}$ = A maximum set of mutually compatible activities in $S_{i,j}$ and $c_{i,j} = |A_{i,j}|$

If $a_{i+1} \in A_{i,j}$, then $c_{i,j} = c_{i,i+1} + c_{i+1,j} + 1$

If $a_{i+2} \in A_{i,j}$, then $c_{i,j} = c_{i,i+2} + c_{i+2,j} + 1$

If $a_{i+3} \in A_{i,j}$, then $c_{i,j} = c_{i,i+3} + c_{i+3,j} + 1$

$\vdots$

If $a_{j-1} \in A_{i,j}$, then $c_{i,j} = c_{i,j-1} + c_{j-1,j} + 1$

$c_{i,j}$ is maximum of these

Recurrence for Activity-Selection

$S_{i,j}$ = Set of activities that start after activity a_i finishes and that finish before activity a_j starts.

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$c_{i,j} = |A_{i,j}|$

Recurrence for Activity-Selection

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$c_{i,j} = |A_{i,j}|$

$$c_{i,j} = \begin{cases} & \\ & \end{cases}$$

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$c_{i,j} = |A_{i,j}|$

$$c_{i,j} = \begin{cases} \;, & \text{if } i = j \\ & \end{cases}$$

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$$c_{i,j} = \begin{cases} \quad , & \text{if } i = j \\ \quad , & \text{if } i \neq j \end{cases}$$

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$c_{i,j} = |A_{i,j}|$

$$c_{i,j} = \begin{cases} \quad , & \text{if } S_{i,j} = \varnothing \\ \quad , & \text{if } S_{i,j} \neq \varnothing \end{cases}$$

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Recurrence for Activity-Selection

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Implementing DP will now take $O(n^3)$ time. Can we do better?

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We do NOT need go over all $a_k \in S_{i,j}$. We can make a GREEDY choice.

Greedy Choice for Activity-Selection

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Which activity is definitely a part of $A_{i,j}$?

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Some possible intuitive choices:

- Activity that finishes first. ✓ ← We will design a greedy algorithm based on this.
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Proving Correctness of Greedy Choice

i	0	1	2	3	4	5	6	7	8	9	10	11	12
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Consider an $A_{0,12}$, say $\{a_2, a_4, a_9, a_{11}\}$, not containing a_1.

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Why a_1 will definitely be a part of some $A_{0,12}$?

Consider an $A_{0,12}$, say $\{a_2, a_4, a_9, a_{11}\}$, not containing a_1.

But a_2 can be replaced with a_1 to produce another $A_{0,12}$.

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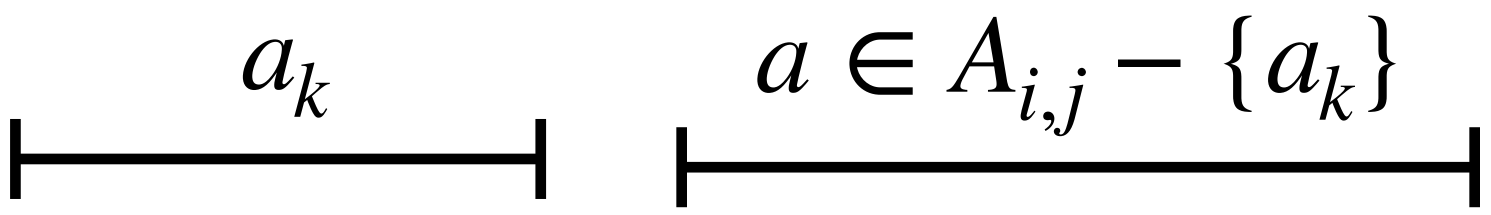

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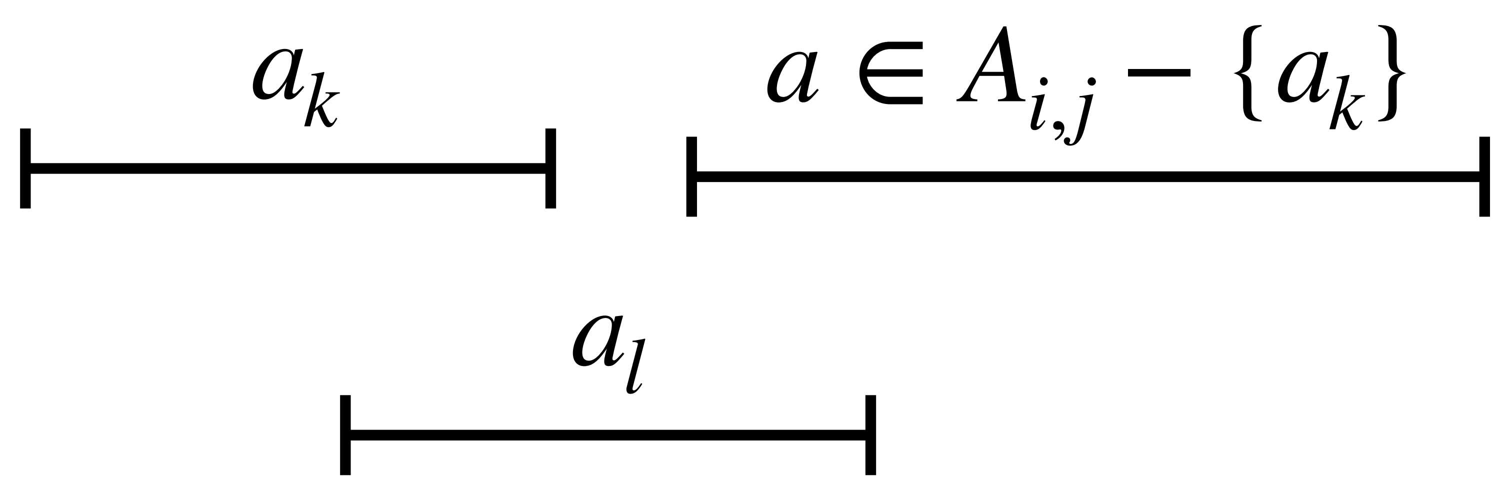

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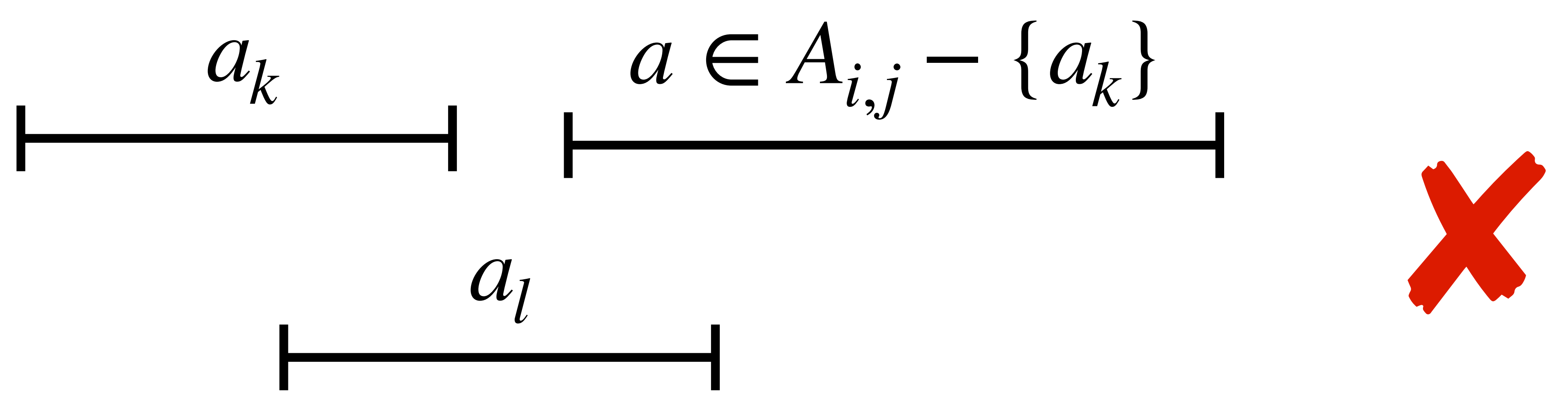

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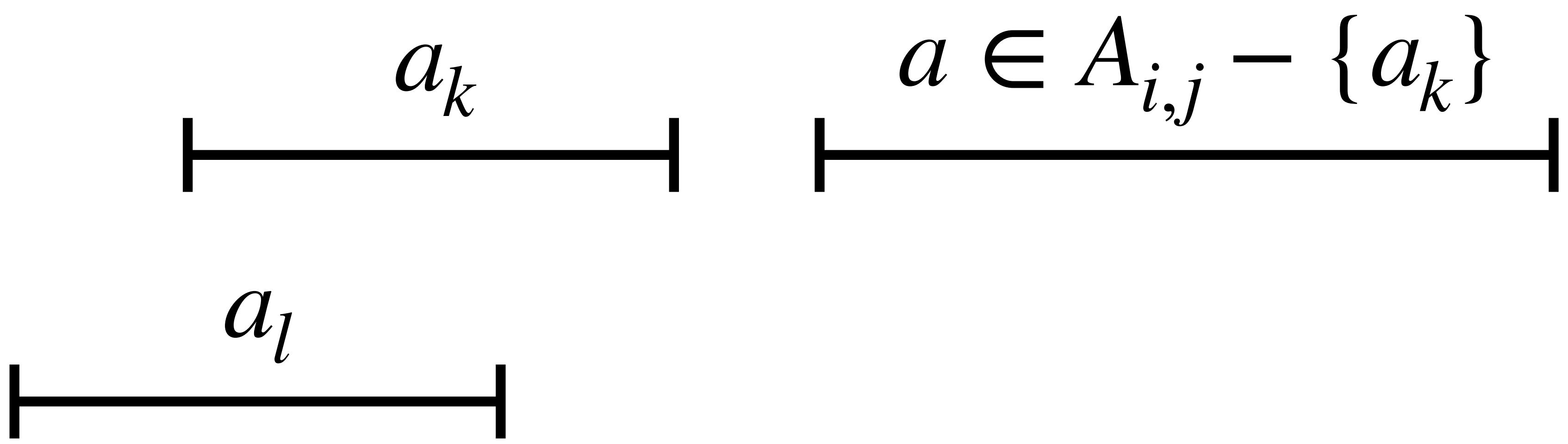

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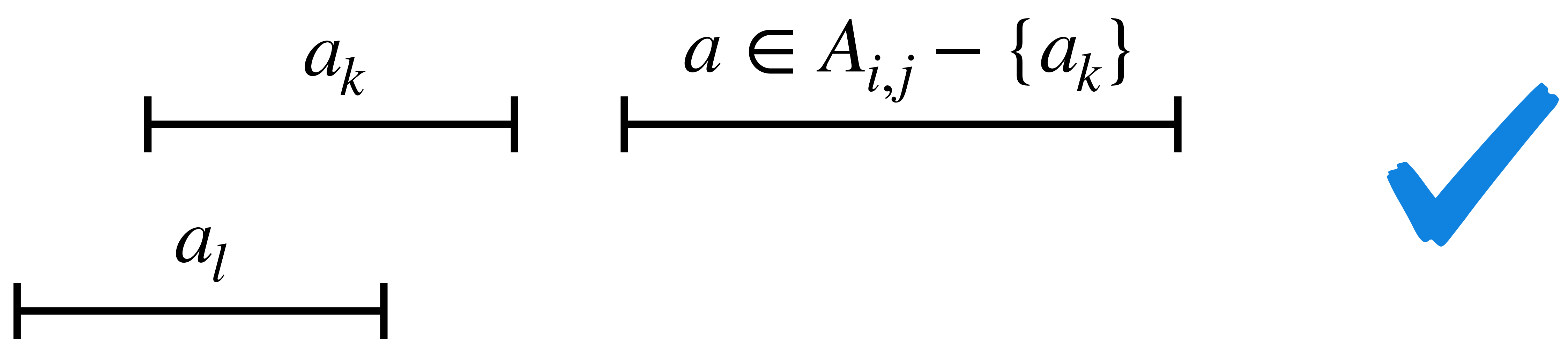

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